5/3/17 Lecture 10 outline

• Born approximation: a t-channel amplitude has $\mathcal{M}_{non-rel} \approx -\int d^3\vec{r}e^{-(\vec{p}'-\vec{p})\cdot\vec{r}}V(\vec{r})$. For e.g. $e^- + e^- \to e^- + e^-$ the tree-level amplitude in the t channel gives the Coulomb potential. Mention Yukawa's theory of a force mediated by scalar exchange, where now the propagator in the Feynman rule has $1/(t - m_\pi^2)$, where m_π is the scalar mass (the pions in Yukaway's theory of nucleon attraction via pion exchange). Plugging into the Born approximation this gives $V(r) = (g/2M)^2 e^{-m_\pi r}/r$, where the $1/2M$ is from relativistic normalization $\bar{\psi}\gamma^\mu\psi = 2p^\mu$ and hence $\psi^\dagger\psi = 2E$ instead of $\langle\psi|\psi\rangle = 1$.

• Fermi's Golden Rule (actually, it was first derived by Dirac, but Fermi used it a lot): transition rate $= (2\pi/\hbar)|\mathcal{M}|^2$ (phase space factors). Recall $d^3\vec{n} = (V/(2\pi)^2)d^3\vec{p}$. The V factors will cancel out. But we have seen that the Lorentz invariant version is $dLIPS = \prod_i d^3p_i/(2\pi)^3(2E_i)$. The $2E$ can here be given a hand-waiving (a rigorous procedure gives the same answer) justification because the states are normalized with $\psi^\dagger\psi = \bar{\psi}\gamma^0\psi = 2E$ as you saw on a HW. So get

$$\prod_i \frac{1}{2E_i}\prod_f \frac{d^3\vec{p}_f}{(2\pi)^2 2E_f}|\mathcal{M}|^2(2\pi)^4\delta^4(\sum p_i - \sum p_f).$$

• E.g. decay $1 \to 2 + \ldots n$ in rest frame. $d\Gamma = |\mathcal{M}|^2(2\pi)^4\frac{S}{2m_1}\delta^4(p_1 - p_2 - p_3 \ldots p_n)\prod_{j=2}^n \frac{d^3p_j}{(2\pi)^3(2E_j)}$ This is $\sim$ prob / sec. Example of $\pi^0 \to \gamma + \gamma$.

• Now consider scattering cross section for $1+2 \to 3+4+\ldots$. Interaction probability is $dP = dN\sigma/A = nv_{rel}\sigma dt$; cross section σ is number of interactions per unit time per target particle divided by the incident flux. Indeed, the decay is $\sim 1/\tau = \sigma v_{rel}$ so $\sigma \sim 1/v_{rel}$. The Lorentz invariant flux factor we divide by is $4E_1E_2|\vec{v}_1 - \vec{v}_2| = 4E_1E_2|\vec{p}_1E_1^{-1} - \vec{p}_2E_2^{-1}| = 4|E_2\vec{p}_1 - E_1\vec{p}_2| = 4\sqrt{(p_1\cdot p_2) - m_1^2m_2^2}$. E.g. for $2 \to 2$ scattering in CM frame get $\sqrt{(p_1\cdot p_2)^2 - m_1^2m_2^2} = (E_1+E_2)|\vec{p}_1|$ so

$$d\sigma = \frac{|\mathcal{M}|^2 S}{4(E_1+E_2)|\vec{p}_1|}\frac{1}{(2\pi)^2}\frac{\delta(E_1+E_2-E_3-E_4)}{(2E_3)(2E_4)}p^2dpd\Omega.$$

Doing the dp integral using the $\delta(E)$ gives finally

$$\frac{d\sigma_{CM}}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{s}\frac{p_f}{p_i}|\mathcal{M}|^2.$$

In a frame where instead $\vec{p}_2 = 0$ (”lab frame”) one similarly finds (see our textbook)

$$\frac{d\sigma_{lab}}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{E_3}{m_pE_1}\right)^2|\mathcal{M}|^2.$$

• Can recover the Rutherford scattering formula as an approximation, in the limit where quantum effects are negligible (tree level) and proton recoil can be neglected so the electron is non-relativistic. Find for the spin averaged amplitude in this limit (i.e. $E_1 = E_3 = E \sim m_e \ll m_p$ and $p_1 = p_3 = p$ so $t = (p_1 - p_3)^2 = -2p^2(1-\cos\theta) = -4p^2\sin^2(\theta/2)$) find $\langle|\mathcal{M}|^2\rangle \approx m_p^2 m_e^2 e^4/p^4\sin^4(\theta/2)$ and then, writing in terms of $E_K \approx p^2/2m_e$, reproduce the classical Rutherford scattering formula (lab frame, treating the proton as at rest):

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{1}{m_p + E_1 - E_1\cos\theta}\right)^2 \langle|\mathcal{M}|^2\rangle \approx \frac{\alpha^2}{16E_K^2\sin^4(\theta/2)}.$$

ended here

• $P_{L,R} = \frac{1}{2}(1\mp\gamma_5)$, $\mathcal{L} = \bar{\psi}(P_L^2 + P_R^2)(i\not{\partial} - m)\psi = \bar{\psi}_R i\not{\partial}\psi_R + \bar{\psi}_L i\not{\partial}\psi_L - m\bar{\psi}_R\psi_L - m\bar{\psi}_L\psi_R$.

Mass terms mix helicity, massless fermions can have well-defined helicity: $\mathcal{L} = \bar{\psi}(P_L^2 + P_R^2)(i\not{\partial} - m)\psi = \bar{\psi}_R i\not{\partial}\psi_R + \bar{\psi}_L i\not{\partial}\psi_L - m\bar{\psi}_R\psi_L - m\bar{\psi}_L\psi_R$.

• QED: seems the electron is a Dirac Fermion, with both helicities, since it is massive. Actually, the electron is chiral; this is seen when E+M are unified with the weak force. All of the known fermions of the SM are chiral. Not known if any fundamental (elementary) Dirac fermions exist in nature. Nothing forbids it, but since they can have large mass, unprotected by any symmetry, they are expected to be extremely massive if they exist, unless there is fine tuning. Actually, as far as we know, there is fine tuning.

• The large s limit.

• Helicity.

• Left and right-handed chiral fermions. Mention again SM Fermions, and that they're actually all chiral.

• Parity and its violation in β decays: into the looking glass.

• CP and CPT and Feynman's story.

• Start isospin. History: nuclear physics has a symmetry that rotates proton into neutron. Sounds weird (they have different electric charges), but the point is that the strong force, i.e. $SU(3)_C$ doesn't distinguish between ps and ns, though the electroweak force does. $SU(2)_I$ representations, for p and n.